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MBU-003-1164003

Seat No. ________

M. Sc. (Sem. IV) (CBCS) Examination

April / May - 2018

Mathematics - 4003

(Number Theory - II)

Faculty Code : 003

Subject Code : 1164003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are five questions in this paper

(2) Each question carries 14 marks

(3) All questions are compulsory

1 Fill in the blanks : (Each question carries two marks)

(a) If $\frac{a}{b}$ and $\frac{c}{d}$ are consecutive Farey fractions in the nth row and $\frac{a}{b}$ is less than $\frac{c}{d}$ then $\frac{a}{b}$ and $\frac{a+c}{b+d}$ are consecutive Farey fractions in the .... th row.

(b) If $\frac{a}{b}$ and $\frac{c}{d}$ are consecutive Farey fractions in the nth row then $\left|\frac{a}{b}-\frac{a+c}{b+d}\right| \le$ ....

(c) If the simple continued fraction expansion of θ is finite then θ must be a .... number.

(d) If θ is an irrational and $\frac{a}{b}$ is a rational number such that $b > 0$ and $\left|\theta - \frac{a}{b}\right| < \frac{1}{2b^2}$ then $\frac{h_n}{k_n}$ = ... for some n.

(e) If continued fraction expansion of an irrational θ is periodic and θ' lies between–1 and 0 then continued fraction expansion of θ is ....

(f) If θ is an irrational, $\frac{a}{b}$ is a rational number such that $\left|\theta - \frac{a}{b}\right| < \left|\theta - \frac{h_n}{k_n}\right|$ for some $n \ge 0$ then b is greater than ....

(g) The Diophantine equation $ax + by = c$ has a solution if and only if .... divides c.

2 Attempt any **two** of the following :

(a) Write the statement of Hurwitz theorem and prove it using Farey fraction method. **7**

(b) Prove that π is irrational using elementary method. **7**

(c) Prove that for each $n > 0$ there is a polynomial $f_n(x)$ of degree n, leading coefficient 1 and with integer coefficients such that $f_n(2\cos\theta) = 2\cos n\theta$. **7**

3 All are compulsory :

(a) State and prove the necessary and sufficient condition under which the continued fraction expansion of a quadratic irrational is purely periodic. **6**

(b) If θ is irrational and $\theta = \langle a_0, a_1, \ldots\ldots\ldots..a_n, \ldots\ldots\ldots \rangle$ then prove that $k_n < \theta_n k_{n-1} + k_{n-2} < k_{n+1}$ for all $n \ge 0$. **4**

(c) Prove that $15x^2 - 7y^2 = 9$ has no solutions in integers. **4**

OR

3 All are compulsory :

(a) Suppose θ is irrational and $\frac{a}{b}$ is a rational number such that $\left|\theta - \frac{a}{b}\right| < \frac{1}{2b^2}$ then prove that $\frac{a}{b} = r_n$ for some n. **7**

(b) Find general solutions (if any) of the following Diophantine equations : **4**

(i) $2x + 9y = 11$

(ii) $100x + 101y = 2018$

(c) Find the value of following continued fractions : **4**

(i) $\langle 1,1,1,1,1 \rangle$

(ii) $\langle 0,2,2,2,2,2,..... \rangle$

4 Attempt any **two** of the following :

(a) Suppose θ is an irrational number whose continued fraction expansion is periodic. Prove that θ is quadratic irrational. **7**

(b) Prove that there are infinitely many positive integers n such that $1+2+3+....+n=m^2$ for some integer m. **7**

(c) If $\frac{h_j}{k_j}$ denotes the jth convergent of an irrational number θ then prove that for all $n \geq 1$. **7**

(i) $|\theta k_n - h_n| < |\theta k_{n-1} - h_{n-1}|$

(ii) $\left|\theta - \frac{h_n}{k_n}\right| < \left|\theta - \frac{h_{n-1}}{k_{n-1}}\right|$

5 Do as directed : (Each question carries two marks)

(a) Give the definition of Diophantine equation.

(b) Express $\frac{2018}{17}$ as a simple continued fraction.

(c) Write down the values of $\frac{h_o}{k_o}, \frac{h_1}{k_1}$ for the continued fraction $\langle 1,1,1,1,.... \rangle$.

(d) Write down all the Farey fractions between 0 and 1 in the rows up to 7th row.

(e) Find first four positive solutions of $x^2 - 8y^2 = 1$.

(f) Find three primitive Pythagorean triplets (x, y, z) for which $z > 50$.

(g) Find three positive integers n for which $1+2+3...+n$ is a perfect square.